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Bulk Viscous Bianchi Type-V Cosmological Models with Variable Gravitational and Cosmological Constant

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Abstract This paper deals with Bianchi type-V cosmological models of the universe filled with a bulk viscous cosmic fluid in the framework of general relativity. A new class of exact solutions has been obtained by considering various well established power law relations among scale factor, cosmological and gravitational constants and cosmic time. Some physical and geometrical behaviors of the models have also been discussed. It has been found that all the models are in fair agreement of observational results.

Keywords Bianchi type-V space time · Bulk viscosity · Cosmological and gravitational constant

1 Introduction

Anisotropic Cosmological models play significant role in understanding the behavior of the universe at its early stages of evolution. Modern cosmology is concerned with thorough understanding and explanation of the past history, the present state and future evolution of the universe. Recent cosmological observations support the existence of an anisotropic phase that approaches to isotropic one [1]. The geometry of anisotropic cosmological models belong to either Bianchi type I–IX or Kantowski-Sachs space-time geometry. Bianchi type-I space-time is considered as simplest generalization of FRW flat space-time and studied by large number of researchers. Among different anisotropic cosmological models Bianchi type-V universe is natural generalization of the open FRW model. Lorentz [2, 3] investigated tilted Bianchi type-V cosmological model with matter and electromagnetic field and in higher dimensions. A large number of authors have studied Bianchi type-V cosmological model in different contexts [4–13]. Singh and Chaubey [14] have considered a Bianchi

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type-V universe initially for self consistent system of gravitational field with a binary mixture of perfect fluid and dark energy given by a cosmological constant. Further they have studied the evaluation of a homogeneous anisotropic universe filled with viscous fluid, in the presence of cosmological constant Λ [15]. Singh et al. [16] have investigated the variation law for Hubble's parameter in a homogeneous and anisotropic Bianchi type-V space time model that yield a constant value of deceleration parameter. Bianchi type-V cosmological models with negative deceleration parameter in scalar tensor theories have been presented by Rao et al. [17–19]. Very recently, Singh and Baghel [20] have studied the Bianchi type-V model with constant deceleration parameter in general relativity and Ram et al. [21] have studied such type of models in Saez-Ballester theory. Barotropic perfect fluid cosmological models in Lyra geometry have been proposed by Bali and Chandnani [22] whereas a model in the presence of massless scalar field with a flat potential having inflationary solution is presented by Reddy [23].

It has been point out in the literature that the dissipative processes may well account for the present high degree of isotropy and also huge value of the number of photons to baryons [24]. The study of role of dissipative effects in the evolution of the universe during early stages has taken considerable interest of researchers as in the early universe viscosity may arise due to various processes such as decoupling of neutrinos during the radiation era, creation of superstring during the quantum era, particle collision involving gravitation, particle creation process and the formation of galaxies [25–28]. It has been suggested that in large class of homogeneous but anisotropic universe, the anisotropy dies away rapidly. The most important mechanism in reducing the anisotropy is neutrinos viscosity at temperature just above 10^{10} K. It is important to develop a model of dissipative cosmological processes in general, so that one can analyze the overall dynamics of dissipation without getting lost in the details of complex processes. The cosmological models in the presence of bulk viscosity have been studied by a number of authors [29–41]. Very recently Singh and Kale [42] have discussed anisotropic bulk viscous cosmological models with variable G and Λ.

The aforesaid survey of literature clearly indicate that there has been considerable interest in study of spatially homogeneous and anisotropic cosmological models and hence it is worthwhile to study anisotropic viscous Bianchi type-V cosmological models with variable G and Λ.

2 Field Equations

The Einstein field equations with cosmological constant may be written as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G T_{ij} + \Lambda g_{ij}. \tag{1}$$

The energy momentum tensor of cosmic fluid take the form

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij}. \tag{2}$$

Here ρ is the energy density, p represents equilibrium pressure. The flow vector u_i is satisfying the relation $u_i u^i = 1$.

The line element for Bianchi type-V space time metric is given by

$$ds^2 = dt^2 - R_1^2(t) dx^2 - R_2^2(t) e^{2\alpha x} dy^2 - R_3^2(t) e^{2\alpha x} dz^2. \tag{3}$$

The Einstein's field equation (1) for the space-time metric (3) yields following equations

$$\frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3}+\frac{\dot{R}_3}{R_3}\frac{\dot{R}_1}{R_1}-\frac{3\alpha^2}{R_1^2}=8\pi G\rho+\Lambda, \tag{4}$$

$$\frac{\ddot{R}_1}{R_1}+\frac{\ddot{R}_2}{R_2}+\frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2}-\frac{\alpha^2}{R_1^2}=-8\pi Gp+\Lambda, \tag{5}$$

$$\frac{\ddot{R}_2}{R_2}+\frac{\ddot{R}_3}{R_3}+\frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3}-\frac{\alpha^2}{R_1^2}=-8\pi Gp+\Lambda, \tag{6}$$

$$\frac{\ddot{R}_1}{R_1}+\frac{\ddot{R}_3}{R_3}+\frac{\dot{R}_1}{R_1}\frac{\dot{R}_3}{R_3}-\frac{\alpha^2}{R_1^2}=-8\pi Gp+\Lambda, \tag{7}$$

$$\frac{2\dot{R}_1}{R_1}-\frac{\dot{R}_2}{R_2}-\frac{\dot{R}_3}{R_3}=0. \tag{8}$$

By combining (4)–(7) one can easily obtain the continuity equation as

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)+\rho\frac{\dot{G}}{G}+\frac{\dot{\Lambda}}{8\pi G}=0. \tag{9}$$

The energy-momentum conservation equation $T^{ij}_{;j}=0$ suggests

$$\dot{\rho}+3(\rho+p)H=0. \tag{10}$$

From (9) and (10), we have

$$\rho\frac{\dot{G}}{G}+\frac{\dot{\Lambda}}{8\pi G}=0, \tag{11}$$

where the Hubble parameter H is given by

$$H=\frac{1}{3}\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right).$$

The physical quantities of observational interest are the expansion scalar θ, shear scalar σ^2, the relative anisotropy and deceleration parameter q which are defined by

$$\theta=3H=\frac{\dot{V}}{V}\quad (V=R_1R_2R_3) \tag{12}$$

$$\sigma^2=\frac{1}{2}\left[(\sigma_1^1)^2+(\sigma_2^2)^2+(\sigma_3^3)^2+(\sigma_4^4)^2\right], \tag{13}$$

where the components of shear tensor (σ_i^j) are given by

$$\sigma_1^1=\frac{1}{3}\left(\frac{2\dot{R}_1}{R_1}-\frac{\dot{R}_2}{R_2}-\frac{\dot{R}_3}{R_3}\right)=0,$$

$$\sigma_2^2=\frac{1}{3}\left(\frac{2\dot{R}_2}{R_2}-\frac{\dot{R}_1}{R_1}-\frac{\dot{R}_3}{R_3}\right),$$

$$\sigma_3^3 = \frac{1}{3}\left(\frac{2\dot{R}_3}{R_3} - \frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2}\right),$$

$$\sigma_4^4 = 0,$$

$$\text{relative anisotropy} = \frac{\sigma^2}{\rho}, \tag{14}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{15}$$

3 Cosmological Solutions

It can be easily seen that we have five independent equations (4)–(8) with seven unknowns R_1, R_2, R_3, ρ, p, G and Λ. In order to obtain complete set of exact solutions, we require two more physically plausible relations among the variables.

Equations (5) and (7) suggest a relation between scale factors R_2 and R_3 as

$$\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = \frac{k}{(R_2R_3)^{3/2}}. \tag{16}$$

Using (8) one can easily obtain the relation among the scale factors as

$$R_1^2 = R_2R_3. \tag{17}$$

Assuming relation between the scale factor and cosmic time t as

$$R_3 = t^n, \tag{18}$$

(16) and (17) yield

$$R_1 = t^n[k_1 + k_2t^{-3n+1}]^{1/3}, \tag{19}$$

$$R_2 = t^n[k_1 + k_2t^{-3n+1}]^{2/3}. \tag{20}$$

Considering equation of state

$$p = \gamma\rho, \quad 0 \le \gamma \le 1, \tag{21}$$

(4), (5), (10), (18)–(21), give following form of the energy density, gravitational and cosmological constant.

$$\rho = \frac{c}{t^{3n(1+\gamma)}U^{(1+\gamma)}}, \tag{22}$$

$$G = \frac{t^{3n(1+\gamma)}U^{(1+\gamma)}}{8\pi c(1+\gamma)}\left[\frac{2n}{t^2} + \frac{6nk_2(1-3n)}{3t^{3n+1}U} + \frac{4k_2^2(1-3n)^2}{9t^{6n}U^2} - \frac{2\alpha^2}{t^{2n}U}\right], \tag{23}$$

$$\Lambda = \left[\frac{3n^2}{t^2} - \frac{2n}{(1+\gamma)t^2} + \left(\frac{\gamma}{1+\gamma}\right)\frac{6nk_2(1-3n)}{3t^{3n+1}U} + \left(\frac{\gamma-1}{\gamma+1}\right)\frac{2k_2^2(1-3n)^2}{9t^{6n}U^2} - \left(\frac{3\gamma+1}{\gamma+1}\right)\frac{\alpha^2}{t^{2n}U}\right], \tag{24}$$

with $U = k_1 + k_2t^{-3n+1}$.

The geometrical quantities of observational interest and deceleration parameter have the following expressions in terms of cosmic time t

$$\theta = \frac{3nk_1 + k_2 t^{-3n+1}}{k_1 t + k_2 t^{-3n+2}}, \tag{25}$$

$$\sigma^2 = \frac{k_2^2(1-3n)^2}{9t^{6n}[k_1 + k_2 t^{-3n+1}]^2}, \tag{26}$$

$$\text{relative anisotropy} = \frac{k_2^2(1-3n)^2}{9Ct^{3n(1-\gamma)}[k_1 + k_2 t^{-3n+1}]^{1-\gamma}}, \tag{27}$$

$$q = -1 + 3\left\{\frac{k_1 + k_2(2-3n)t^{-3n+1}}{[3nk_1 + k_2 t^{-3n+1}]} - \frac{k_2(1-3n)t^{-3n+1}[k_1 + k_2 t^{-3n+1}]}{[3nk_1 + k_2 t^{-3n+1}]^2}\right\}. \tag{28}$$

In this case the geometry of the model of the universe is given by

$$ds^2 = dt^2 - t^{2n}\left(\left(k_1 + k_2 t^{-3n+1}\right)^{2/3} dx^2 + \left(k_1 + k_2 t^{-3n+1}\right)^{4/3} e^{2\alpha x} dy^2 + e^{2\alpha x} dz^2\right). \tag{29}$$

It can be easily seen that the energy density, pressure, bulk viscosity are decreasing with evolution of the universe. The deceleration parameter q suggests that for all values of $n > 1$ the model presents accelerating expansion of the universe. It can be seen from the space-time metric (29) that for large value of cosmic time this anisotropic cosmological model asymptotically approaches to isotropic model. In this case shear as well as relative anisotropy vanish and expansion stops. All the results of the model presented in this section are in favor of the report of present days observations.

4 Cosmological Models with Bulk Viscosity

The energy momentum tensor of cosmic fluid in the presence of bulk viscosity takes the form

$$T_{ij} = (\rho + p + \Pi)u_i u_j - (p + \Pi)g_{ij}. \tag{30}$$

Here Π stands for bulk viscous stress.

The Einstein's field (1) for the space-time metric (3) yields following equations

$$\frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} + \frac{\dot{R}_3}{R_3}\frac{\dot{R}_1}{R_1} - \frac{3\alpha^2}{R_1^2} = 8\pi G\rho + \Lambda, \tag{31}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_2}{R_2} - \frac{\alpha^2}{R_1^2} = -8\pi G(p + \Pi) + \Lambda, \tag{32}$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2}{R_2}\frac{\dot{R}_3}{R_3} - \frac{\alpha^2}{R_1^2} = -8\pi G(p + \Pi) + \Lambda, \tag{33}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1}\frac{\dot{R}_3}{R_3} - \frac{\alpha^2}{R_1^2} = -8\pi G(p + \Pi) + \Lambda, \tag{34}$$

$$\frac{2\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = 0. \tag{35}$$

By combining (31)–(34) one can easily obtain continuity equation as

$$\dot{\rho} + (\rho + p + \Pi)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) + \rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{36}$$

In the presence of bulk viscosity the energy-momentum conservation equation $T^{ij}_{;j} = 0$ suggests

$$\dot{\rho} + 3(\rho + p + \Pi)H = 0, \tag{37}$$

and hence (36) yields

$$\rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{38}$$

In order to obtain complete set of exact solutions the number of unknowns must be equal to the number of equations, therefore we require three more physically plausible relations amongst the variables. In the following sections we will consider power law relation between scale factors, gravitational and cosmological constants.

4.1 Case I: Power Law Relation Between Average Scale Factor and Cosmic Time

Now considering

$$V = R_1 R_2 R_3 = t^n. \tag{39}$$

Equation (17) suggests

$$V = R_1^3 = t^n. \tag{40}$$

Using relation (40) in to (32)–(34), we obtain

$$R_2 = \frac{1}{k_2} t^{n/3} e^{\frac{k_1}{(n-1)t^{(n-1)}}}, \tag{41}$$

$$R_3 = k_2 t^{n/3} e^{\frac{-k_1}{(n-1)t^{(n-1)}}}. \tag{42}$$

Here k_1 and k_2 are constants of integration.

Considering equation of state $p = \gamma\rho$ and assuming a well accepted power law relation between gravitational constant G and cosmic time t [37, 43–45] as

$$G = G_0 t^m, \tag{43}$$

(31)–(34), (40)–(43) yield exclusive expression for the energy density, bulk viscosity and cosmological constant in terms of cosmic time t as follows,

$$\rho = \left(\frac{a_0}{t^{m+2}} - \frac{a_1}{t^{\frac{2n+3m}{3}}} - \frac{a_2}{t^{2n+m}}\right), \tag{44}$$

$$\Pi = -\left(\frac{b_0}{t^{m+2}} + \frac{b_1}{t^{\frac{2n+3m}{3}}} + \frac{b_2}{t^{2n+m}}\right), \tag{45}$$

$$\Lambda = \left(\frac{c_0}{t^2} - \frac{c_1}{t^{(2n/3)}} - \frac{c_2}{t^{2n}}\right), \tag{46}$$

where

$$a_0 = \frac{n^2 a^2}{12\pi G_0(m+2)}, \qquad a_1 = \frac{3n\alpha^2}{4\pi G_0(3m+2n)}, \qquad a_2 = \frac{nk_1^2}{4\pi G_0(m+2n)}$$

$$b_0 = \frac{(1+\gamma)n^2a^2 - na^2(m+2)}{12\pi G_0(m+2)}, \qquad b_1 = \frac{3m\alpha^2 - (1+3\gamma)n\alpha^2}{4\pi G_0(3m+2n)},$$

$$b_2 = \frac{mk_1^2 + (1-\gamma)nk_1^2}{4\pi G_0(m+2n)}$$

$$c_0 = \frac{mn^2a^2}{3(m+2)}, \qquad c_1 = \frac{9m\alpha^2}{(3m+2n)} \quad \text{and} \quad c_2 = \frac{mk_1^2}{(m+2n)}.$$

The quantities of observational interest have the following expressions in terms of time t

$$\theta = \frac{n}{t}, \tag{47}$$

$$\sigma^2 = \frac{k_1^2}{t^{2n}}, \tag{48}$$

$$\text{relative anisotropy} = \frac{k_1^2 t^m}{[a_0 t^{2n-2} - a_1 t^{4n/3} - a_2]}, \tag{49}$$

$$q = -1 + \frac{3}{n}. \tag{50}$$

In this case the Bianchi type-V space-time metric takes the form

$$ds^2 = dt^2 - t^{2n/3}\left(dx^2 + e^{2\alpha x}\left\{e^{\frac{2k_1}{(n-1)t^{(n-1)}}}dy^2 + e^{\frac{-2k_1}{(n-1)t^{(n-1)}}}dz^2\right\}\right). \tag{51}$$

Now, we are interested in study of the variation in bulk viscosity coefficient (ξ) and temperature (T) with respect to cosmic time.

For the full causal non equilibrium thermodynamics the causal evolution equation for bulk viscosity is

$$\Pi + \tau\dot{\Pi} = -\xi\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) - \frac{\varepsilon\tau\Pi}{2}\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right). \tag{52}$$

Here T ≥ 0 is the absolute temperature, ξ is the bulk viscosity coefficient, τ denotes the relaxation time for transient bulk viscous effects. When $\varepsilon = 0$, (52) reduces to evolution equation for truncated theory. For full causal theory $\varepsilon = 1$ and the non-causal theory (Eckart's theory) has $\tau = 0$ [30].

Maartens [31] has pointed out that the Gibb's integrability condition suggest if the equation of state for pressure is barotropic ($p = p(\rho)$) then the equation of state for temperature should be barotropic ($T = T(\rho)$) and it may be expressed as

$$T \propto \exp\int \frac{dp(\rho)}{\rho + p(\rho)}$$

which with the help of (21) reduces to

$$T = T_0 \rho^{\frac{\gamma}{(1+\gamma)}}, \tag{53}$$

where T_0 stands for a constant.

Here for sake of simplicity, we will discuss only the effect on bulk viscosity with evolution of the universe in non-causal theory ($\tau = 0$). In this case (52) takes the form

$$\Pi = -\xi \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) \tag{54}$$

with the help of (40)–(42), (45) and (54) we have the relation between bulk viscosity coefficient and cosmic time t as

$$\xi = \left(\frac{b_0}{t^{m+1}} + \frac{b_1}{t^{\frac{2n+3m-3}{3}}} + \frac{b_2}{t^{2n+m-1}} \right). \tag{55}$$

The present cosmological model is singular model having accelerated expansion for all $n > 3$.

All the physical and geometrical parameters energy density, equilibrium pressure p, temperature, bulk viscosity coefficient, expansion scalar, shear and relative anisotropy are decreasing with evolution of the universe.

4.2 Case II: Power Law Relation for Gravitational and Cosmological Constants with Scale Factor

Several authors [33, 46–48] have considered a power law relation between scale factor and scalar field ϕ in Brans-Dicke theory. As the gravitational constant G varies as ϕ^{-1} we have considered following relation.

$$G = G_0 R^m \tag{56}$$

and a well accepted relation between scale factor and cosmological constant [49–54]

$$\Lambda = \Lambda_0 R^{-2}. \tag{57}$$

From (32)–(35) and after suitably adjusting the constant of integrations, we obtain

$$R_1 = R_2 = R_3 = R. \tag{58}$$

Now the set of (30)–(33) assumes the form

$$3\left(\frac{\dot{R}}{R}\right)^2 - \frac{3\alpha^2}{R^2} = 8\pi G\rho + \Lambda, \tag{59}$$

$$2\left(\frac{\ddot{R}}{R}\right) + \left(\frac{\dot{R}}{R}\right)^2 - \frac{\alpha^2}{R^2} = -8\pi G\,(p + \Pi) + \Lambda. \tag{60}$$

Equations (56)–(60), (38) along with equation of state $p = \gamma\rho$ for $0 \le \gamma \le 1$ suggest the scale factor, energy density, bulk viscosity, cosmological constant and gravitational constant as follows

$$R = (at + b), \tag{61}$$

where $a=\sqrt{\frac{2\Lambda_0}{3m}+\frac{\Lambda_0}{3}+\alpha^2}$ and b is integration constant.

$$\rho=\frac{2\Lambda_0}{8\pi G_0 m(at+b)^{m+2}}, \tag{62}$$

$$\Pi=\frac{-(1+3\gamma-m)\Lambda_0}{12\pi G_0 m(at+b)^{m+2}}, \tag{63}$$

$$G=G_0(at+b)^m, \tag{64}$$

$$\Lambda=\Lambda_0(at+b)^{-2}. \tag{65}$$

In this case expansion scalar, shear and deceleration parameter are having following values

$$\theta=\frac{a}{(at+b)}, \tag{66}$$

$$\sigma=0, \tag{67}$$

$$q=0. \tag{68}$$

4.2.1 Subcase (i) Behavior of Bulk Viscosity in the Eckart's Theory

In Eckart's non causal theory ($\tau=0$) the evolution equation (52) for bulk viscosity takes the form

$$\Pi=-\xi\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right). \tag{69}$$

With the help of (58), (61), (63), and (69), we have the relations between bulk viscosity coefficient and cosmic time t as

$$\xi=\frac{(1+3\gamma-m)\Lambda_0}{36a\pi G_0 m}\frac{1}{(at+b)^{m+1}}. \tag{70}$$

4.2.2 Subcase (ii) Behavior of Bulk Viscosity in Truncated Theory

It has been already been pointed out that in truncated theory ($\varepsilon=0$), the evolution equation (52) for bulk viscosity reduces to

$$\Pi+\tau\dot{\Pi}=-3\xi H. \tag{71}$$

The following relation

$$\tau=\frac{\xi}{\rho} \tag{72}$$

has been adopted as one way of ensuring that viscous signals do not exceed the speed of light in the truncated theory [55, 56].

By use of (58), (61)–(63), (72) into (71) we get the expression for bulk viscosity coefficient as

$$\xi=\frac{(1+3\gamma-m)\Lambda_0}{4a\pi G_0 m[9-(1+3\gamma-m)(m+2)]}\frac{1}{(at+b)^{m+1}}. \tag{73}$$

4.2.3 *Subcase (iii) Behavior of bulk viscosity in Full causal theory*

The evolution equation (52) for bulk viscosity in full causal theory for $\varepsilon = 1$ may be written as

$$\Pi + \frac{\xi}{\rho}\dot{\Pi} = -\xi\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) - \frac{\xi\Pi}{2\rho}\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} - \frac{(1+2\gamma)}{(1+\gamma)}\frac{\dot{\rho}}{\rho}\right). \tag{74}$$

Further, using the expression for scale factors, energy density and bulk viscous stress from (58), (61), (62) and (63) into (74) one can easily obtained the relation between bulk viscosity coefficient (ξ) and cosmic time (t) as

$$\xi = \frac{(1+\gamma)(1+3\gamma-m)\Lambda_0}{2a\pi G_0 m[(1+3\gamma-m)\{(1+\gamma)(2m+4-3a)-(1+2\gamma)(m+2a)\}+18a(1+\gamma)]} \times \frac{1}{(at+b)^{m+1}}. \tag{75}$$

In this particular model geometry of the universe may be specified by following space-time line element

$$ds^2 = dt^2 - (at+b)^2(dx^2 + e^{2\alpha x}(dy^2 + dz^2)). \tag{76}$$

This cosmological model is a nonsingular model which suggests universe starts from a finite value and starts expanding with uniform rate of expansion. All the physical parameters are in fair agreement of standard observational results.

5 Conclusion

This research paper may be divided mainly into two parts. The first part of the paper consists of a cosmological model in absence of bulk viscosity. The survey of literature suggests that large number of cosmological models have power law and exponential relation between scale factor and cosmic time. However power law relations have been preferred due to certain advantage in favor of cosmological observations. In order to find exact solutions of the set of field equation a power law relation $R_3 = t^n$ is assumed. This model suggests that in the beginning universe was anisotropic and with accelerated expansion of the universe it approaches to isotropic universe. In the second part of this paper the role of bulk viscosity during evolution of the universe is considered. Considering well established power law relation for average scale factor, cosmological constant, gravitational constant and cosmic time, exact solutions for entire set of field equations have been obtained. It has been found that all the physical and geometrical parameters are in fair agreement of observational results.

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References

1. Land, K., Magueijo, J.: Phys. Rev. Lett. **95**, 071301 (2005)
2. Lorenz, D.: Gen. Relativ. Gravit. **13**, 795 (1981)

3. Lorenz-Petzold, D.: Lett. Math. Phys. **10**, 279 (1985)
4. Beesham, A.: Astrophys. Space Sci. **125**, 99 (1986)
5. Banerjee, A., Sanyal, A.K.: Gen. Relativ. Gravit. **20**, 103 (1988)
6. Nayak, B.K., Sahoo, B.K.: Gen. Relativ. Gravit. **21**(3), 211 (1989)
7. Coley, A.A.: Gen. Relativ. Gravit. **22**, 3 (1990)
8. Singh, T., Singh, G.P.: Astrophys. Space Sci. **182**, 189 (1991)
9. Coley, A.A., Dunn, K.: J. Math. Phys. **33**, 1772 (1992)
10. Coley, A.A., Hoogen, R.J.: J. Math. Phys. **35**, 4117 (1994)
11. Nayak, B.K., Sahoo, B.K.: Gen. Relativ. Gravit. **28**, 251 (1996)
12. Pradhan, A., Rai, A.: Astrophys. Space Sci. **291**, 149 (2004)
13. Raj, B., Singh, D.K.: Astrophys. Space Sci. **300**, 387 (2005)
14. Singh, T., Chaubey, R.: Pramana **67**, 415 (2006)
15. Singh, T., Chaubey, R.: Pramana **68**, 721 (2007)
16. Singh, C.P., Shri, R., Zeyauddin, M.: Astrophys. Space Sci. **315**, 181 (2008)
17. Rao, V.U.M., Vinutha, T., Shanthi, V.M.: Astrophys. Space Sci. **314**, 213 (2008)
18. Rao, V.U.M., Vinutha, T., Santhi, V.M., Sireesha, K.V.S.: Astrophys. Space Sci. **317**, 89 (2008)
19. Rao, V.U.M., Vinutha, T., Santhi, V.M., Sreedevi, G.: Astrophys. Space Sci. **317**, 89 (2008)
20. Singh, J.P., Baghel, P.S.: Int. J. Theor. Phys. **48**, 449 (2009)
21. Shri, R., Zeyauddin, M., Singh, C.P.: Pramana **72**, 415 (2009)
22. Raj, B., Chandnani, N.K.: Int. J. Theor. Phys. **48**, 1523 (2009)
23. Reddy, D.R.K.: Int. J. Theor. Phys. **48**, 2036 (2009)
24. Weinberg, S.: Gravitation and Cosmology. Wiley, New York (1972)
25. Misner, C.W.: Astrophys. J. **151**, 431 (1968)
26. Ellis, G.F.R.: In: Sachs, R.K. (ed.) Relativistic Cosmology. Academic, New York (1971)
27. Borrow, J.D., Matzner, R.A.: Mon. Not. R. Astr. Soc. **181**, 719 (1977)
28. Hu, B.L.: In: Fang, L.Z., Ruffini, R. (eds.) Advances in Astrophysics. World Scientific, Singapore (1983)
29. Gron, O.: Astrophys. Space Sci. **173**, 191 (1990)
30. Maartens, R.: Class. Quantum Gravity **12**, 1455 (1995)
31. Maartens, R.: Extended irreversible thermodynamics in relativity. In Maharaj, S. (ed.) Hanno Rund Lecture Notes at University of Natal, Durban, South Africa. arXiv:astro-ph/9609119v1 (18 September 1996)
32. Mak, M.K., Harko, T.: Gen. Relativ. Gravit. **30**, 800 (1998)
33. Singh, G.P., Beesham, A.: Aust. J. Phys. **52**, 1039 (1999)
34. Mak, M.K., Harko, T.: Int. J. Mod. Phys. D **11**, 447 (2000)
35. Singh, T., Singh, G.P., Beesham, A.: Astrophys. Space Sci. **275**, 275 (2001)
36. Pradhan, A., Pandey, H.R.: Int. J. Mod. Phys. D **12**, 941 (2003)
37. Singh, G.P., Deshpande, R.V., Singh, T.: Pramana **63**, 937 (2004)
38. Yadav, M.K., Roy, A., Pradhan, A.: Int. J. Theor. Phys. **46**, 2677 (2007)
39. Calistete, R. Jr., Fabris, J.C., Tossa, J., Zimdhal, W.: Phys. Rev. D. **76**, 103516 (2007)
40. Singh, C.P., Suresh, K.: Int. J. Theor. Phys. **48**, 925 (2008)
41. Pradhan, A.: Commun. Theor. Phys. **51**, 378 (2009)
42. Singh, G.P., Kale, A.Y.: Int. J. Theor. Phys. **48**, 1177 (2009)
43. Kalligas, D., Wesson, P.S., Everitt, C.W.: Gen. Relativ. Gravit. **24**, 3511 (1992)
44. Kalligas, D., Wesson, P.S., Everitt, C.W.: Gen. Relativ. Gravit. **27**, 645 (1995)
45. Beesham, A.: Gen. Relativ. Gravit. **26**, 159 (1994)
46. Luis, O.P.: Astrophys. Space Sci. **112**, 175 (1985)
47. Johri, V.B., Kalyani, D.: Gen. Relativ. Gravit. **26**, 1217 (1994)
48. Azar, E.A., Riazi, N.: Astrophys. Space Sci. **226**, 1 (1995)
49. Ozer, M., Taha, M.O.: Phys. Lett. B **171**, 363 (1986)
50. Ozer, M., Taha, M.O.: Nucl. Phys. **287**, 776 (1987)
51. Vishwakarma, R.G.: Class. Quant. Gravit. **14**, 945 (1997)
52. Chen, W., Wu, Y.S.: Phys. Rev. D **41**, 695 (1990)
53. Abdel Rahman, A.M.: Phys. Rev. D **45**, 3497 (1992)
54. Sahni, A., Starobinsky, A.: Int. J. Mod. Phys. D **9**, 373 (2000)
55. Zakari, M., Jou, D.: Phys. Rev. D **48**, 1597 (1993)
56. Romano, V., Pavon, D.: Phys. Rev. D **50**, U72 (1994)